



ESTIMATING PERIOD JITTER FROM PHASE NOISE

1. Introduction

This application note reviews how RMS period jitter may be estimated from phase noise data. This approach is useful for estimating period jitter when sufficiently accurate time domain instruments, such as jitter measuring oscilloscopes or Time Interval Analyzers (TIAs), are unavailable.

2. Terminology

In this application note, the following definitions apply:

- Cycle-to-cycle jitter—The short-term variation in clock period between *adjacent* clock cycles. This jitter measure, abbreviated here as J_{CC} , may be specified as either an RMS or peak-to-peak quantity.
- Jitter—Short-term variations of the significant instants of a digital signal from their ideal positions in time (Ref: Telcordia GR-499-CORE). In this application note, the digital signal is a clock source or oscillator. *Short-term* here means phase noise contributions are restricted to frequencies greater than or equal to 10 Hz (Ref: Telcordia GR-1244-CORE).
- Period jitter—The short-term variation in clock period over *all* measured clock cycles, compared to the average clock period. This jitter measure, abbreviated here as J_{PER} , may be specified as either an RMS or peak-to-peak quantity. This application note will concentrate on estimating the RMS value of this jitter parameter. The illustration in Figure 1 suggests how one might measure the RMS period jitter in the time domain. The first edge is the reference edge or trigger edge as if we were using an oscilloscope.

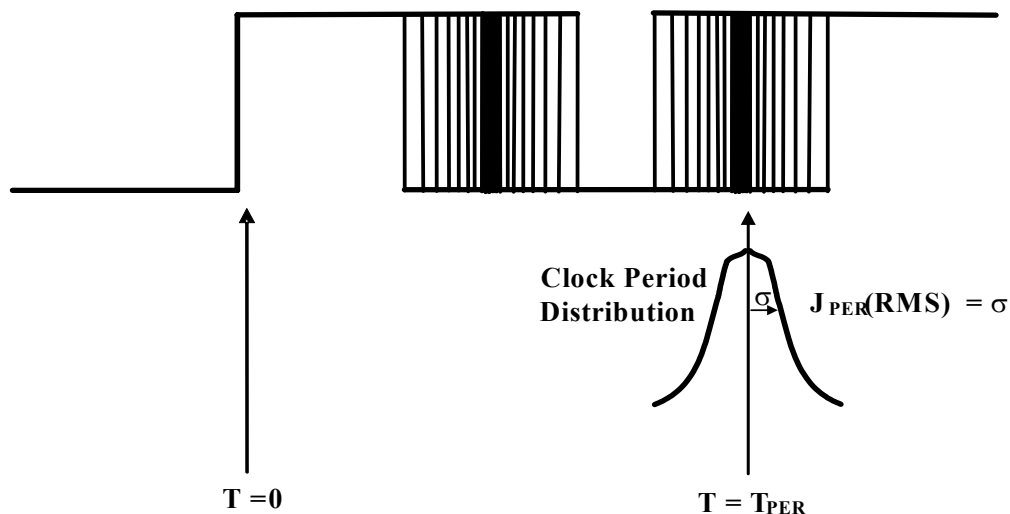


Figure 1. RMS Period Jitter Example

- Phase jitter—The integrated jitter (area under the curve) of a phase noise plot over a particular jitter bandwidth. Phase noise data may be recorded as either SSB phase noise $L(f)$ in dBc/Hz or phase noise spectral density $S_{\phi}(f)$ in rad^2/Hz where:

$$L(f) \equiv \frac{S_{\phi}(f)}{2}$$

RMS phase jitter may be expressed in units of dBc, radians, time, or Unit Intervals (UI).

3. Basic Approach

By definition, period jitter compares two similar instants in time of a clock source such as two successive rising edges or two successive falling edges. Since the two instants are separated in time by approximately one period, it is reasonable to expect that higher frequency jitter components will contribute more to period jitter than lower-frequency jitter components ($f \ll 1/T$.)

A number of authors, e.g., Drakhlis (2001), have derived the basic relationship between J_{PER} (rms) and phase noise spectral density $S_{\phi}(f)$. It can be shown that

$$\Delta\phi_{RMS}^2 = 4 \int_0^{\infty} S_{\phi}(f) \sin^2(\pi f \tau) df$$

and, therefore:

$$J_{PER}(rms) = \frac{T_0}{2\pi} \sqrt{\Delta\phi_{RMS}^2}$$

In short, period jitter is integrated similarly to phase jitter but with a frequency weighting factor of $4\sin^2(\pi f \tau)$. See "Appendix A—Derivation" on page 5.

There are practical limits to integrating phase noise from $f = 0$ to ∞ . Phase noise is measured from a low-frequency limit, f_L , to a high-frequency limit, f_H . In practice, the minimum and maximum phase noise offset frequencies are determined by the measurement system bandwidth.

Phase noise integration using the \sin^2 weighting factor is not typically sensitive to contributions from offset frequencies well below the half-carrier frequency. Therefore, the choice of f_L in theoretical period jitter calculations is generally not critical. For the purposes of this application note, the minimum offset frequency for phase noise measurements is 10 Hz, coinciding with the usual dividing line between wander and jitter.

The period jitter \sin^2 weighting factor reaches its first maximum at the half-carrier frequency and becomes periodic thereafter. It is clear that phase noise in the vicinity of this offset will make a significant contribution to the period jitter.

The maximum offset frequency for the purposes of integration is determined by the bandwidth of interest. One practical high-frequency limit is to set f_H equal to the half-carrier frequency. For an example, see Underhill and Brown (2004). Another, more conservative, approach is to integrate out to the carrier frequency where the \sin^2 weighting factor reaches its first null. This method appears to correlate well with at least one popular Time Interval Analyzer or TIA, presumably due to aliasing components above $f_C/2$. See Smith (2006).

However, in many high-frequency applications, phase noise equipment cannot measure phase noise all the way out to either the half-carrier or carrier frequencies. Provided that the phase noise data reaches the phase noise floor at its highest measured offset, the phase noise floor may be extended from $f_{H, MEASURED}$ to f_H . This is the estimation approach adopted in this application note.

4. Example

Consider the example in Figure 2, which represents the SSB phase noise in dBc/Hz measured for a 160 MHz oscillator.

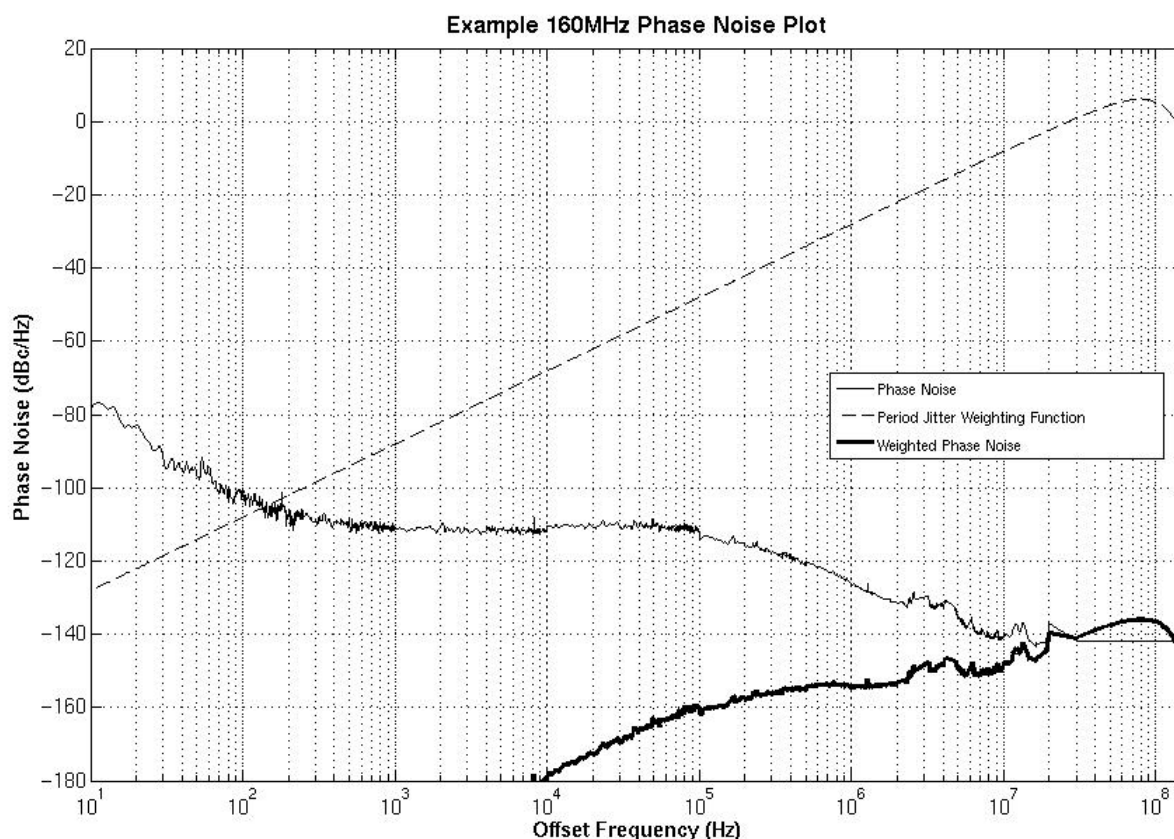


Figure 2. 160 MHz Oscillator SSB Phase Noise

For this particular plot, there are three curves illustrated as follows:

- **Phase Noise**—This is the upper solid curve going from -78.32 dBc/Hz @10 Hz on the left down to -142 dBc @80 MHz on the right.
- **Period Jitter Weighting Function**—This is the dashed line depicting the $4\sin^2(\pi f\tau)$ weighting factor in dB. It is predominately a $+20$ dB/dec sloped line until reaching a peak at the half-carrier frequency where it turns over to a null at the carrier frequency.
- **Resulting Phase Noise**—This is the lower, thicker solid curve representing the summation of the phase noise in dBc/Hz with the Period Jitter Weighting Function in dB.

The particular phase noise instrument that took the original phase noise data had a default maximum offset frequency of 20 MHz when measuring clocks less than 250 MHz. For the purpose of estimating the period jitter, the "ultimate noise floor" of the original data set has been extended from the last data point at 20 MHz out to the carrier frequency of 160 MHz.

In this example, there are no discrete components or spurs. However, if there had been any significant spurs in the vicinity of the half-carrier frequency, they would need to be weighted similarly as described in "Appendix B—Calculating with Spurs Included" on page 7.

By numerical integration, we can determine that the integrated phase noise under the entire SSB phase noise curve from 10 Hz to 160 MHz yields a total phase noise power = -54.46 dBc. This "brick wall" integration is equivalent to wideband RMS phase jitter of 2.663 ps or 0.00268 radians.

By contrast, if we weight $L(f)$ using the period jitter weighting function and integrate the resulting curve over 10 Hz to 160 MHz, we obtain a total phase noise power = -56.84 dBc, equivalent to RMS period jitter of 2.025 ps or 0.00204 radians; so, we can estimate the period jitter as being roughly 2 ps RMS, based solely on the available phase noise measurement. This estimate can be correlated with measurements from complementary time domain equipment when available. See Smith (2006) for example.

The estimation process may be simplified further by treating the period jitter weighting function as if it was a single-pole, high-pass filter function out to $fc/2$ only. As noted in Figure 2, the J_{PER} weighting function, based on the $4\sin^2(\pi f\tau)$ weighting factor in dB, looks like a +20 dB/dec sloped line until it gets close to the half-carrier frequency, where it starts to curve down. If one instead approximates the weighting function as $4(\pi f\tau)^2$ in dB, this yields a straight +20 dB/dec line. The maximum or intercept at the half-carrier frequency is, therefore,

$10\log(4(\pi(fc/2)\tau)^2) = 10\log(\pi^2) = 9.94$ dB or approximately 10 dB. Integrating the weighted phase noise in this fashion, one obtains -57.62 dBc, equivalent to RMS period jitter of 1.849 ps or 0.00186 radians. In this particular example, the simplified “single pole filter” weighting out to the half carrier frequency resulted in a roughly 9% lower estimate. This is a less conservative approach, which may still yield a sufficiently accurate estimate depending on the application.

Since this approach does not directly measure period jitter in the time domain, we cannot make definitive statements about the peak-to-peak values of the period jitter. However, if other evidence suggests that the edge jitter or period jitter distribution is Gaussian, we can apply the usual statistical estimates.

5. Summary

This application note has reviewed how RMS period jitter may be estimated from phase noise data. The key points to keep in mind are as follows:

- Period jitter is dominated by high-frequency phase noise.
- Channel bandwidth plays a determining role in the apparent jitter observed.
- Spurs contribute very little to the period jitter unless they are large and located in the vicinity of the half-carrier frequency.

Generally speaking, hybrid crystal oscillators, such as the Si5xx series devices, have excellent period jitter due to their relatively low noise floor.

6. References

Drakhlis, Boris, "Calculate Oscillator Jitter By Using Phase-Noise Analysis," *Microwaves & RF*, Jan. 2001 pp. 82-90, 157.

Underhill, M. J. and P. J. Brown, "Estimation of Total Jitter and Jitter Probability Density Function from the Signal Spectrum", *Proc. 18th EFTF (European Time and Frequency Forum)*, University of Surrey, Guildford, UK, 5-7 April 2004.

Smith, Kevin G., "Practical Issues Correlating Theoretical Period Jitter vs. T1A Measured Period Jitter", Austin Conference on Integrated Systems & Circuits, 2006.

APPENDIX A—DERIVATION

The following calculations are based on the zero crossings derivation of Drakhlis (2001).

Let Δt represent the jitter accumulated in one period.

$$\Delta t = \frac{T_0}{2\pi}(\varphi(t_1) - \varphi(t_2))[\text{seconds}]$$

Where: T_0 = nominal period.

t_1 = first zero crossing.

t_2 = second zero crossing.

$$\langle \Delta t^2 \rangle = \left(\frac{T_0}{2\pi}\right)^2 \langle (\varphi(t_1) - \varphi(t_2))^2 \rangle$$

We can drop the $T_0/2\pi$ factor for simplicity and add it back in later.

$$\langle \Delta \varphi^2 \rangle = [\langle \varphi(t_1)^2 \rangle - 2 \langle \varphi(t_1) \rangle \times \langle \varphi(t_2) \rangle + \langle \varphi(t_2)^2 \rangle][\text{radians}^2]$$

Since $\varphi(t)$ is stationary:

$$\langle \varphi(t_1)^2 \rangle = \langle \varphi(t_2)^2 \rangle = \langle \varphi(t)^2 \rangle$$

Per Parseval's theorem:

$$\langle \varphi(t)^2 \rangle = \int_0^{\infty} S \varphi(f)$$

Further:

$$\langle \varphi(t_1) \times \varphi(t_2) \rangle = R_{\varphi}(t_2 - t_1)$$

$$\langle \varphi(t_1) \times \varphi(t_2) \rangle = R_{\varphi}(\tau)$$

Where $R_{\varphi}(\tau)$ is the autocorrelation of $\Phi(f)$, and $\tau = T_0$

The phase noise autocorrelation equals the cosine transform of the phase noise.

$$\langle \varphi(t_1) \times \varphi(t_2) \rangle = \int_0^{\infty} S \varphi(f) \cos(2\pi f \tau) df$$

$$\Delta \varphi_{RMS}^2 = 2 \int_0^{\infty} S \varphi(f) (1 - \cos(2\pi f \tau)) df$$

$$\Delta \varphi_{RMS}^2 = 2 \int_0^{\infty} S \varphi(f) \times 2 \sin^2(\pi f \tau) df$$

$$\Delta \varphi_{RMS}^2 = 4 \int_0^{\infty} S \varphi(f) \times \sin^2(\pi f \tau) df$$

$$\Delta\phi_{RMS}^2 = 2x \left[4 \int_0^{\infty} L(f) \sin^2(\pi f \tau) df \right]$$

Now, convert back to time units:

$$\Delta t_{RMS}^2 = \left(\frac{T_0}{2\pi} \right)^2 \Delta\phi_{RMS}^2 = 2 \frac{T_0^2}{\pi^2} \int_0^{\infty} L(f) \sin^2(\pi f \tau) df = 2 \frac{T_0^2}{\pi^2} \int_{f_L}^{f_U} L(f) \sin^2(\pi f \tau) df$$

where f_L and f_U are the practical lower and upper frequency integration limits.

$$J_{PER\,RMS} = \sqrt{\Delta t_{RMS}^2}$$

APPENDIX B—CALCULATING WITH SPURS INCLUDED

Spurs that are small and few in number and do not contribute significantly to phase jitter may typically be omitted from period jitter calculations. This is especially true for low offset frequency spurs. Each significant spur contribution should be accounted for separately as for the weighted i^{th} spur below:

$$\Delta\phi_{RMS_i}^2 = 8L(f_i) \times \sin^2(\pi f_i \tau)$$

$$\Delta\tau_{RMS_i}^2 = \left(\frac{T_0}{2\pi}\right)^2 8L(f_i) \sin^2(\pi f_i \tau) = \frac{2T_0^2}{\pi^2} L(f_i) \sin^2(\pi f_i \tau)$$

Each spur's contributions are then added in sum square fashion as follows:

$$\Delta t_{RMS_TOTAL}^2 = \Delta t_{RMS_NOISE}^2 + \Delta t_{RMS_1}^2 + \dots + \Delta t_{RMS_i}^2 + \dots + \Delta t_{RMS_N}^2$$

or:

$$\Delta t_{RMS_TOTAL}^2 = \Delta t_{RMS_NOISE}^2 + \sum_{i=1}^N \Delta t_{RMS_i}^2$$

$$J_{PER_TOTAL}(rms) = \sqrt{\Delta t_{RMS_TOTAL}^2}$$

CONTACT INFORMATION

Silicon Laboratories Inc.

4635 Boston Lane
Austin, TX 78735
Tel: 1+(512) 416-8500
Fax: 1+(512) 416-9669
Toll Free: 1+(877) 444-3032
Email: VCXOinfo@silabs.com
Internet: www.silabs.com

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