Integrated Phase Noise

1. Introduction

Phase noise is commonly used to describe the performance of oscillators and is a measure of the power spectral density of the phase angle. Noise in the phase angle of a sinusoid is visible on the power spectral density of the carrier sinusoid as a spread of the true carrier tone. Phase noise in the frequency domain is equivalent to jitter in the time domain. In this application note, we will refer to phase noise power integrated over a bandwidth as phase jitter. Phase jitter, when calculated from phase noise, is an RMS quantity.

In addition to noise, there are other repeating phenomena that generate additional tones, usually much lower than the carrier. Such tones are labeled as "spurs." The word "spur" comes from the word "spurious", which means "not the original." Spurs are signals close to the primary/carrier frequency but that are not the primary signal. Such signals can be problematic for some applications but may have no effect on other systems. Typically, the term "spurs" refers to those that are not harmonics of the carrier signal; so, many devices specify non-harmonic spurious power separately. Spurs require additional consideration when calculating the integrated jitter.

2. Power Spectral Density Measurements

Phase noise is typically plotted on a per Hertz basis. This means that the power level is considered to be uniform across a 1 Hz brick-wall bandwidth (also called the resolution bandwidth). Unfortunately, it would take a very long time to sweep a 1 Hz band-pass filter in 1 Hz steps across the entire frequency spectrum in order to obtain the phase noise data. Instead, a larger resolution bandwidth is used depending on the frequency range offset from the carrier. For example, a 1 MHz resolution bandwidth may be used for the offset range from 10 to 100 MHz, thereby reducing the number of frequency bins from 90 million to 90.

Once the frequency bins have been measured, the noise in a 1 Hz band can be calculated by subtracting 10 dB for every decade frequency drop from the resolution bandwidth to 1 Hz. For example, a 1 MHz resolution bandwidth bin would be scaled by:

10 x Log10(1 MHz/1Hz) = 60 dB.

Using a smaller resolution bandwidth, a second pass across the frequency range may be made in order to improve the spurious signal’s height above the noise floor.

3. Calculating Phase Jitter from Phase Noise

To obtain the time integral of the phase jitter, one can simply take the frequency integral of the phase noise. This is the direct result from Parseval's Theorem, which, simply stated, says that the time integral of the square of a signal is equal to the frequency integral of the square of its Fourier Transform.

Since real phase noise data cannot be collected across a continuous spectrum, a summation must be performed in place of the integral. The summation can be performed using a rectangular or trapezoidal approximation or even Simpson's rule, depending on personal preference. Lastly, phase noise data is typically plotted on a log-log scale, whereas the integration should be performed on a linear-linear scale. The conversion from d/b/a/Hz to seconds/Hz is derived in " Appendix—Mathematical Treatment of Spurs" on page 5 and is restated here:

$$\frac{10^{\mathcal{L}_{dB}(f)/20}}{\pi f_0} = T_j$$

3.1. Example without Spurs

Figure 1 shows a traditional phase noise plot for a 1066 MHz carrier for which we can calculate the integrated jitter. Figure 2 shows the actual timing jitter versus frequency by employing the result of the appendix. This figure also provides a better visual representation of the dominant noise source across the frequency band. Figure 3 expands the first 10 MHz of the plot; the first pass of the rectangular approximation is also shown.

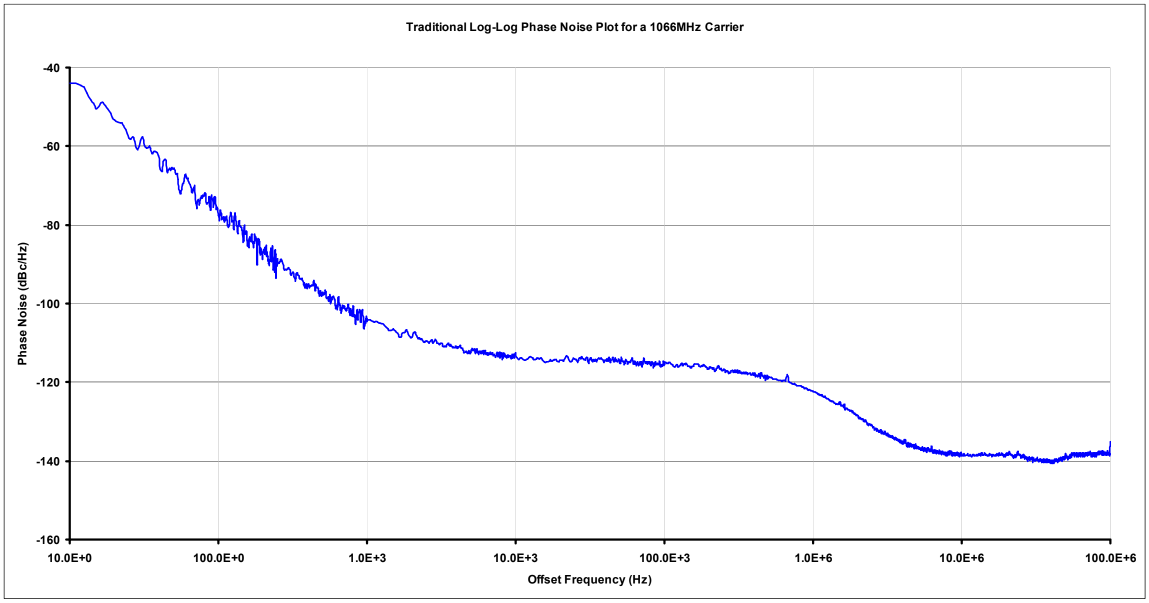

Figure 1. Traditional Log-Log Phase Noise Plot

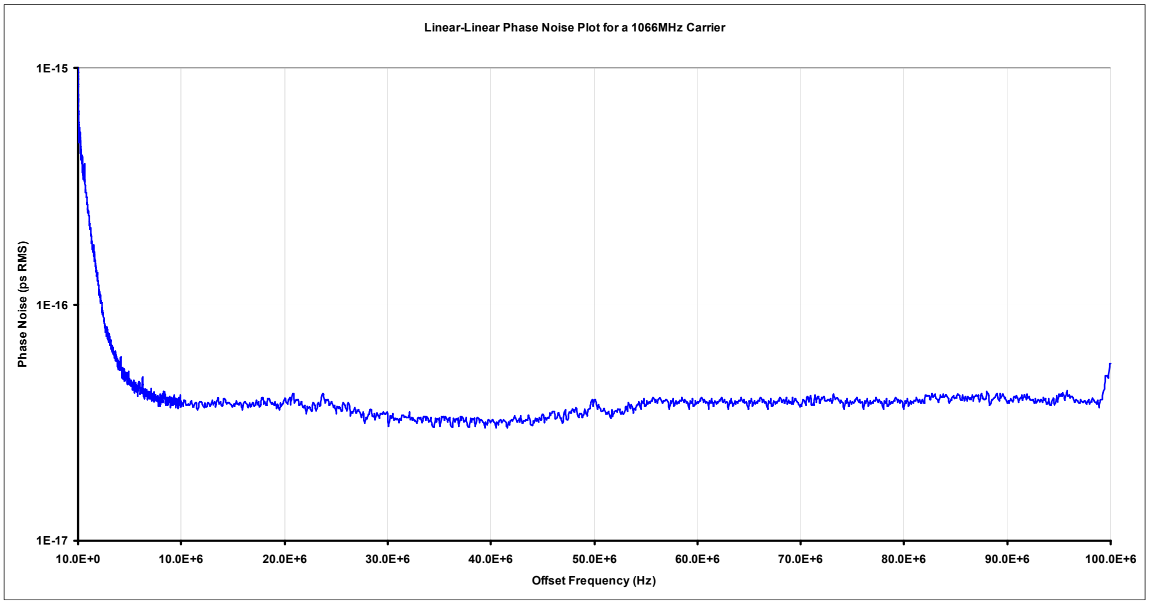

Figure 2. Linear-Linear Phase Noise Plot

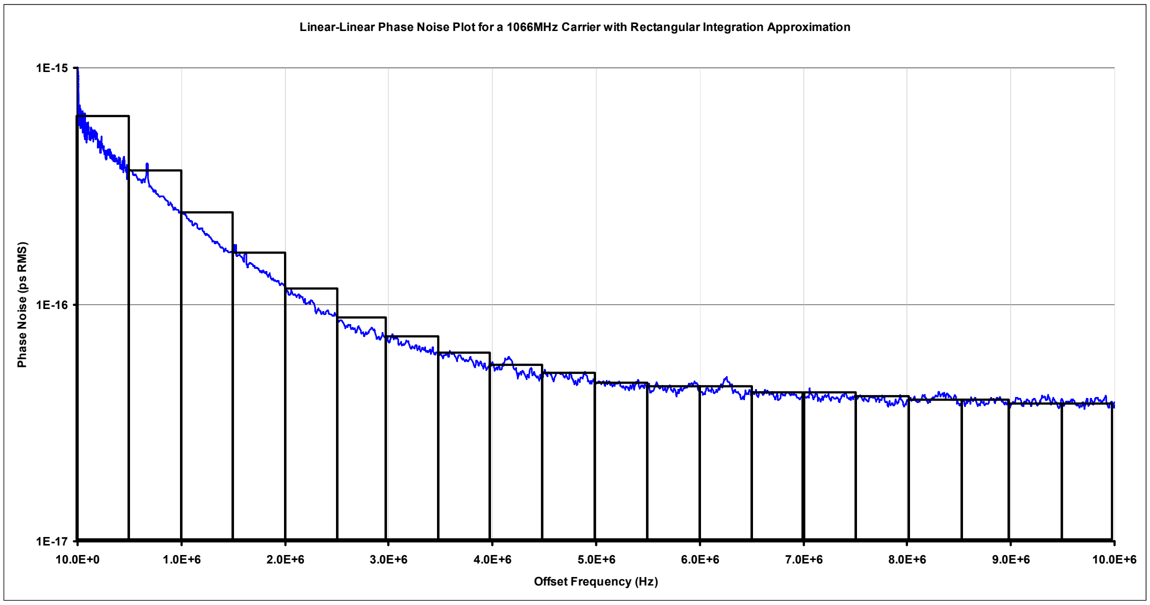

Figure 3. Expanded Scale for the Linear-Linear Phase Noise Plot

Using Figure 3, the summation proceeds as follows. Each rectangle has 5 MHz of bandwidth so the area of each rectangle is:

Area = Height x Width

Area = L(f) x 5 MHz

Since each amplitude, L(f), is an RMS the summation should be performed as the square root of the sum of the squares.

4. Spurs

Additional consideration must be made when spurious power is visible on a phase noise spectrum. The primary concern arises from the excessive width given to spurious tones because of the measurement technique. As discussed above, a large resolution bandwidth would cause the spur to look just as wide, so that when the rectangular approximation is made, too many rectangles fill in the spur. Instead, a spur should be treated as a sinusoidal modulation and be calculated separately from the noise and then root-sum-square added back into the total jitter. The phase noise of the modulation signal can be ignored since its spreading is well correlated to the modulation tone and should be well within the resolution bandwidth that originally captured it.

This technique works well enough but may overestimate the jitter if there are many well-correlated spurs. The phase noise data has already discarded the spurious signals correlation; so, it is not possible to know the true RMS due to multiple well-correlated spurs. Fortunately, the typical RMS value will not be altered much by secondary spurs that are lower than the primary.

4.1. Example with Spurs

Figure 4 shows a traditional phase noise plot for a 1066 MHz carrier for which we can calculate the integrated jitter now with a spur at 30 MHz. The calculation proceeds just as before, with an exception for each spur. Applying a 5 MHz width to the rectangle beginning at 30 MHz greatly exaggerates the jitter caused by the spurious tone (Figure 5). Furthermore, it would not be correct to simply narrow the width of the rectangle to fit the curve because doing so assumes that all of the power in the spur is uncorrelated. The correct way to treat this spur is to assume that it is a pure sinusoidal tone and added into the total integrated jitter as follows:

$$(\text{Total Jitter})^2 = (\text{Integrated Noise Jitter})^2 + (\text{Spurious Tonal Jitter})^2$$

$$\text{Spurious Tonal Jitter} = \frac{10^{\mathcal{L}_{spur}/20}}{\pi f_0}$$

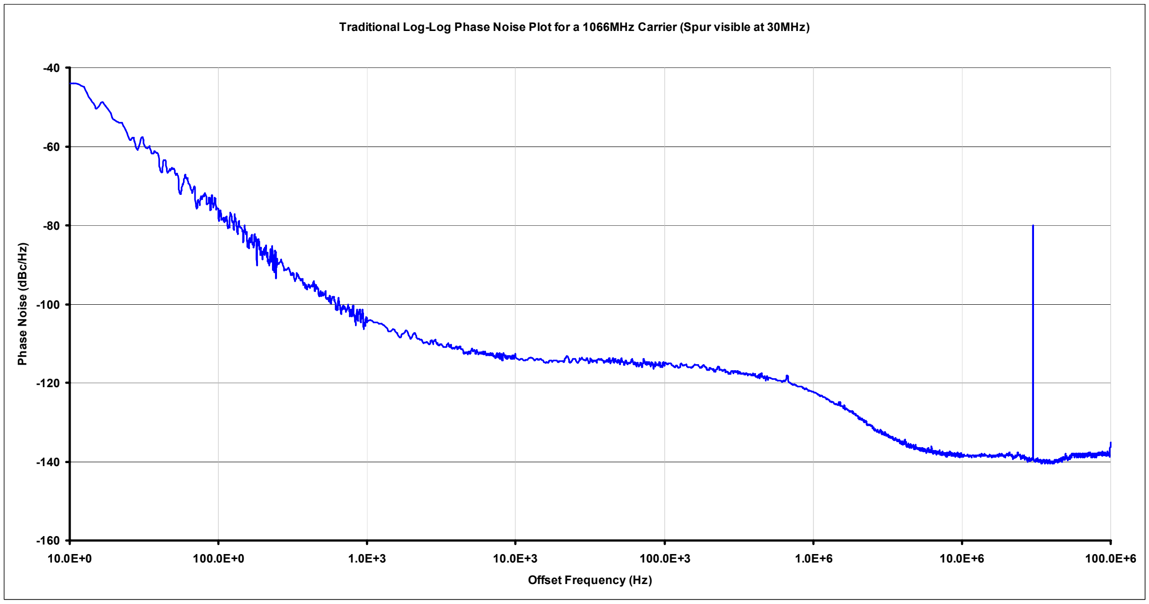

Figure 4. Traditional Phase Noise Plot with Spur at 30 MHz

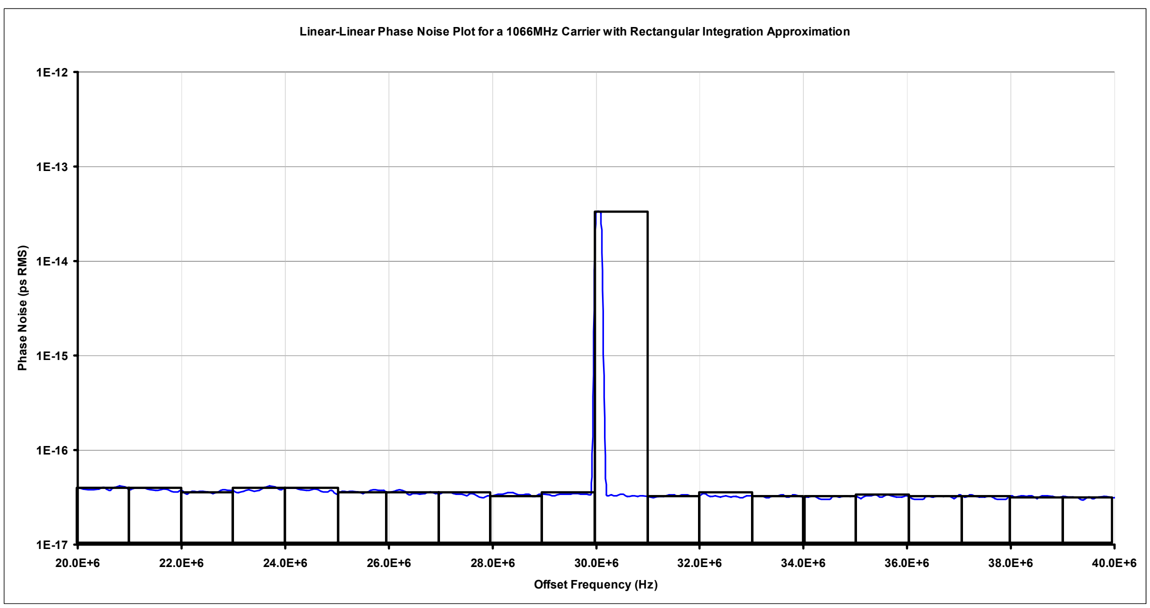

Figure 5. Exaggeration of the 30 MHz Spur by the Rectangular Approximation

5. Conclusion

A method for calculating phase jitter from measured phase noise data is shown. The method also considers spurious signals and shows how to incorporate these into the phase jitter calculations.

APPENDIX—MATHEMATICAL TREATMENT OF SPURS

Spurs in the frequency domain are a direct result of amplitude and/or phase/frequency modulation. A general expression for a signal of interest is as follows:

$$V(t) = [V_P + V_n(t)] \times \cos(2\pi f_0 + \Phi_n t)$$

Where $V_n(t)$ is amplitude noise, and $\Phi_n(t)$ is phase noise.

Generally, it is not possible to write a time domain function for noise, but it is useful to understand how a system responds to sinusoids instead.

Let:

$$V(t) = [V_P + Vn(t)] \times \cos(2\pi f_o t + \Phi_p \sin(2\pi f_m t))$$

So that we are substituting for $\Phi_n(t)$ a sinusoidal signal.

Expanding V(t) into a Fourier Series and then grouping the terms allows us to identify the equation as follows:

$$V(t) = [V_P + Vn(t)] \times [J_0(\Phi_p)\cos(wt) - J_1(\Phi_p)\cos(2\pi(f_0 - f_m)t)^{\cdots}$$
$$+ J_1(\Phi_p)\cos(2\pi(f_0 - f_m)t) - J_2(\Phi_p)\cos(2\pi(f_0 - f_m)t)\ldots]$$

Where J_n are the Bessel Functions of order n.

Next, we assume that the phase noise signal is much less than a period. This allows for the following simplifications:

$$J_1(\Phi_p) \approx (\Phi_p)/2$$

$$J_0(\Phi_p) \approx 1$$

Next, we consider the definition of a phase noise plot:

$$\mathcal{L}(f) = \frac{\text{Power Density (one PM sideband)}}{\text{Power (total signal)}}$$

Then, we substitute our simplified signal:

$$\mathcal{L}(f) = \left(\frac{\Phi_p}{2}\right)^2 = \left(\frac{J_1(\Phi_p)}{J_0(\Phi_p)}\right)^2$$

Since Φp was in radians, we convert to time as follows:

$$\mathcal{L}(f) = (\pi f_0 T_j)^2$$

or in dB

$$\mathcal{L}(f) = 20\log(\pi f_0 T_j)$$

This last equation gives us a way of calculating the peak jitter due to a sinusoidal excitation. Rearranging the terms gives the following:

$$\frac{10^{\mathcal{L}_{dB}(f)/20}}{\pi f_0} = T_j$$

Document Change List

Revision 0.1 to Revision 0.2

- Updated "1. Introduction" on page 1.
- Changed “3.Calculating Period Jitter from Phase Noise” on page 1 to "3. Calculating Phase Jitter from Phase Noise" on page 1.
- Updated "5. Conclusion" on page 4.

CONTACT INFORMATION

Silicon Laboratories Inc.

4635 Boston Lane
Austin, TX 78735
Tel: 1+(512) 416-8500
Fax: 1+(512) 416-9669
Toll Free: 1+(877) 444-3032

Email: Clockinfo@silabs.com
Internet: www.silabs.com

SILICON LABORATORIES